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The hinge lines of non-cylindrical folds

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ABSTRACT

Hinge lines are loci of high curvature points on folded surfaces. They are significant geometrical features of geological folds, and the arrangement of hinge lines constructed for the surface serves to characterize important aspects of the fold pattern. Since the current definition of hinge line is only appropriate for cylindrical folds, we propose a new definition for use with folds of general shape. Like the concept of ridge lines used in differential geometry, the new definition uses the lines of curvature (principal curvature trajectories) as a reference frame for comparing curvatures across the surface. A hinge line passes through points of extreme principal curvature magnitude observed along the corresponding principal curvature trajectory. Two types of hinge lines are defined and methods for constructing hinge lines are suggested.

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1. Introduction

Recently developed surveying methods allow the mapping of folded geological surfaces in three dimensions. 3D seismic reflection methods are employed to map subsurface structures, whereas GPS and laser scanning methods are being increasingly used to survey well-exposed structural surfaces (Bergbauer and Pollard, 2004; Pearce et al., 2006). Whilst these data provide the potential for gaining new insights concerning the process of folding (Pollard and Fletcher, 2005), they also highlight the inadequacy of a number of existing methods of geometrical analysis. The latter were mainly devised to cater for folds outcropping at the surface where information on the folded surface is little more than two-dimensional.

New 3D methods for describing and analysing folded surfaces, founded on the concepts of differential geometry, are being devised to address the above problems (Pollard and Fletcher, 2005; Lisle and Toimil, 2007; Mynatt et al., 2007). For example, approaches have been proposed for dissecting a general folded surface into individual folds, and for distinguishing antiformal and synformal folds (Lisle and Toimil, 2007).

Hinge lines, the subject of this paper, are key geometrical features of folds. The patterns of hinge lines are important in relation to the structural location of hydrocarbon fields (e.g. Al-Mahmoud et al., 2009), fracture prediction in hydrocarbon reservoirs (e.g. Stephenson et al., 2007), prediction of the direction of subsurface elongation of ore-bodies (e.g. Duuring et al., 2007), the analysis of structures produced by multiple folding events (Ramsay, 1967), and to folding processes in shear zones (Ghosh et al., 1999; Alsop and Carreras, 2007) or around diapirs (Jackson et al., 1990).

In the present paper, we examine the existing definition of the hinge line. The term, hinge line, refers to the locus of points of maximum curvature on the folded surface (Fig. 1). Sets of hinge lines drawn on a folded surface serve to illustrate the folding pattern, refolded geometries, and relationships between different fold sets. However, we discover that the existing definition of hinge line is inadequate for general use, and therefore a new definition based on the concepts of differential geometry is proposed. In devising the new definition, our aim is to provide a conceptual framework for the practical construction of hinge lines on folded surfaces whilst honouring the essential meaning of the existing term. Fortunately, we have been assisted in our aims by current research in non-geological fields where it is found useful to draw feature lines along the main creases of a curved surface, and to use the pattern of such lines as "shape fingerprints" of the surface. For

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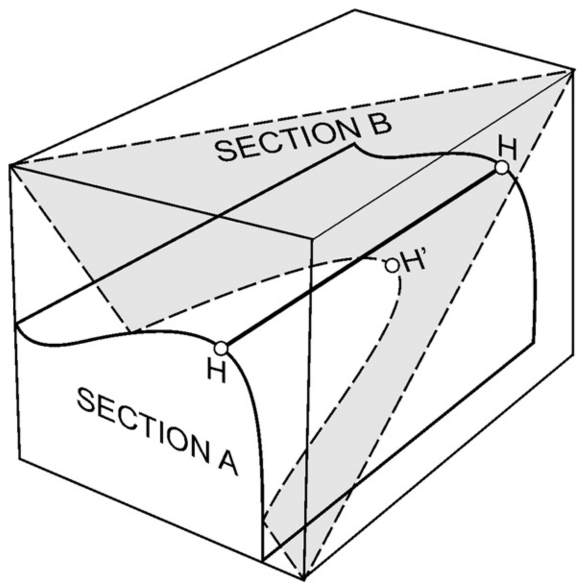

Fig. 1. Cylindrically folded surfaces possess true profile planes, e.g. section A. Points of maximum curvature of the folded surface observed in the profile plane, H, lie on the fold's hinge line, H–H. In general, points of greatest curvature observed on other oblique section planes, e.g. H′ on section B, do not lie on the hinge line.

this reason, literature relating to face recognition (Gordon and Vincent, 1992), analysis of medical images (Thurion and Gourdon, 1996), computer-aided design (Lukács and Andor, 1998) and food engineering (Sivertsen et al., 2009) are pertinent to the issues addressed in this paper.

2. Existing definitions of the term hinge line

Formal definitions of the term, fold hinge line, vary, though the consensus view is that it is a line on a folded surface along which the curvature of the surface reaches a local maximum. As such, it separates adjacent regions of lesser curvature called fold limbs.

A historical review of the concept by Wilson and Cosgrove (1982) reports that the definition of hinge line as the locus of points of large curvature has prevailed at least since the early part of the 20th century (e.g. Haug, 1924; Bonte, 1953; Stockwell, 1950; Wilson, 1961). Some authors have defined the term in a more specific sense by limiting its application to cylindrical, or approximately cylindrical, folds (Wegman, 1929; Clark and McIntyre, 1951). Such hinges are straight lines, or nearly so. Fleuty (1964), however, considered this latter usage to be too restrictive, suggesting that for non-cylindrical folds the hinge line can be drawn through points of maximum curvature observed on serial cross-sections through the folded surface. This definition corresponds to that by Turner and Weiss (1963), Ramsay (1967) and Marshak and Mitra (1988) and results in hinges that are not necessarily straight lines.

When applied to folds that do not deviate greatly from the cylindrical type, the above definition of Turner and Weiss (1963) is workable and generally leads to satisfactory results. However, from a theoretical point of view the definition is flawed, and this leads to practical problems in locating the hinges of non-cylindrical folds. The essence of the problem lies in the choice of the orientation of the plane of section used to determine hinge points. It is well known that the curvature and the location of the point of greatest curvature observed on a 2D section are strongly influenced by the orientation of the section plane through a folded surface (Fig. 1). For instance, even in the case of cylindrical folds, the points of maximum curvature observed on an oblique section do not generally lie on the true hinge line of the fold (Schryver, 1966). Fleuty's (1964) suggestion to determine hinge points on serial sections parallel to the fold's profile plane provides no solution to this problem because non-cylindrical folds do not possess profile planes or natural cross-sections.

In summary, current definitions do not permit the drawing of hinge lines on non-cylindrically folded surfaces. This issue presents a real problem for fold analysis, especially as recent technological advances in mapping suggest that non-cylindrical folds are the norm rather than the exception.

3. General definition of hinge line

Cylindrical folds possess natural profile planes; planes perpendicular to the fold axis (generator). On a serial set of such planes, points of maximum curvature can be identified and then linked to form the hinge line. However, this procedure is not possible in the case of non-cylindrical folds because they lack true profile planes for the observation of curvature variations. To overcome this problem, we propose a modified definition of hinge line based on the concept of ridge lines used in the field of differential geometry (e.g. Koenderink, 1990, p. 291). The ridge lines referred to here are not to be confused with ridges in the topographic sense.

At any point, P, on a curved surface the curvature of the surface can be observed in section planes normal to the plane tangential to the surface at P. In general, these values of normal curvatures vary with the direction chosen for the normal section. In fact, these normal curvatures change systematically as the normal section plane is turned, and reach extreme values in two orthogonal directions of the section plane (Fig. 2). These are the principal curvatures k_1 and k_2 at P, where $k_1 > k_2$ and where convex-upward curvature is positive. The principal curvatures are associated with two perpendicular directions in the surface called the principal curvature directions.

Lines of curvature, or principal curvature trajectories, are curves drawn on the surface whose tangents at any point are parallel to one of the principal curvature directions (Fig. 3). Therefore, two sets of principal trajectories, corresponding to k_1 and k_2 directions, respectively, form an orthogonal mesh on the surface. Ridge lines are the locus of points where the k_1 and k_2 reach extreme values along their respective trajectories (Koenderink, 1990; Belyaev and Anoshkina, 2005, p. 50). The curvature trajectories therefore provide a convenient reference frame for the assessment of

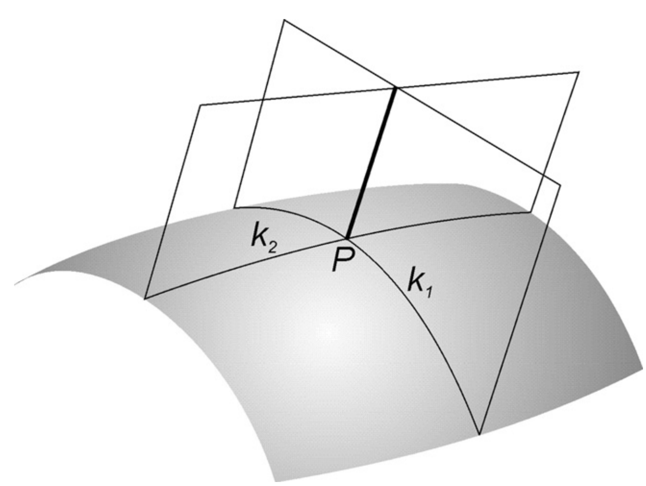

Fig. 2. The definition of principal curvatures, k_1 and k_2, and the principal curvature directions at a point P on a folded geological surface.

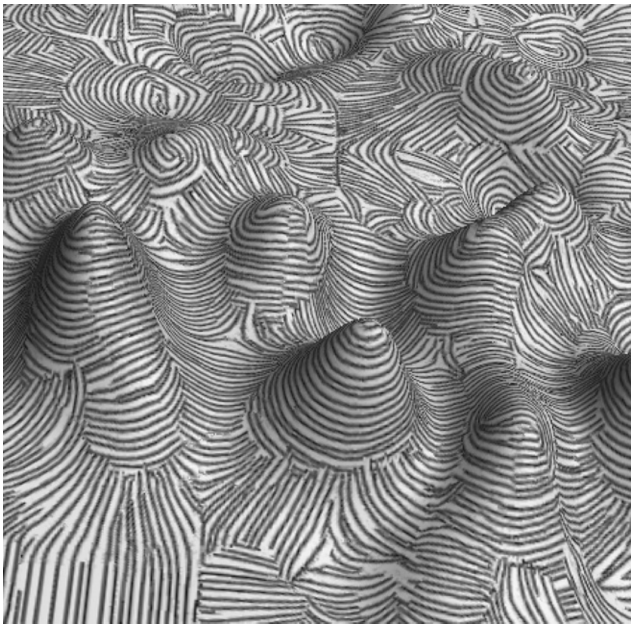

Fig. 3. Example of a folded surface draped with one family of principal curvature trajectories. The zebra stripes are principal curvature trajectories corresponding to principal curvature with the greater absolute value (from Kim et al., 2003, with permission). Strong hinge points are points of maximal absolute curvature along these trajectories.

extreme of curvature, and are a generalization of the function of the profile plane through cylindrical folds. Suppe (1985) uses curvature trajectories to define the term hinge line in relation to cylindrical folds.

We define fold hinge lines as special types of ridge lines. Reference is made to principal curvatures $|k|_{max}$, $|k|_{min}$ labelled according to their absolute magnitudes (i.e. disregarding sign). Two types of hinge lines are distinguished. The first type is the locus of points where $|k|_{max}$ achieves a maximum in absolute value along its own trajectory (Fig. 4). These are named strong hinge lines because they correspond to pronounced, perceptually salient feature lines on the surface. They are the most useful hinge lines for defining the undulations of the surface. Strong hinge lines are antiformal or synformal depending on whether the principal curvature with the greater absolute magnitude is positive or negative, respectively. In other words, strong antiformal and synformal hinges only exist on antiforms and synforms, respectively, i.e., on folds where the mean curvature is positive and negative, respectively (Lisle and Toimil, 2007). The second type of hinge line is defined as the locus of points where $|k|_{min}$ achieves a maximum in absolute value along its trajectory. These so-called weak hinge lines are antiformal or synformal depending on the sign of the principal curvature that has the lower absolute value.

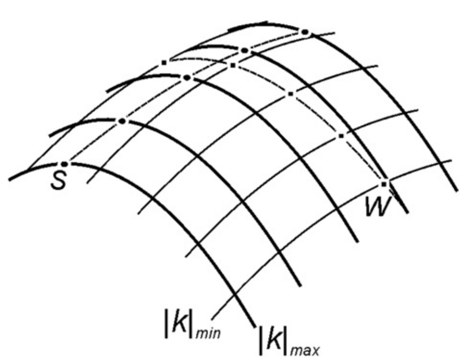

Fig. 4. Principal curvature trajectories (solid) and hinge lines (dashed), Strong hinge points (circles); weak hinge points (squares).

4. Properties of hinge lines

Some properties of hinge lines are worthy of note. They are intrinsic features that have fixed locations within a folded surface, and are unaffected by body rotation of the surface, though the antiformal and synformal labels may interchange. Hinges cannot pass through points on the surface where $|k_1| = |k_2|$. The reason is that at such points, which include umbilical points where $k_1 = k_2$ as well as points on the boundaries of antiformal and synformal folds where the mean curvature is zero (Lisle and Toimil, 2007), are points where curvature trajectories for $|k|_{max}$ and $|k|_{min}$ cannot be distinguished.

In the case of cylindrical folds, the hinge lines constructed in the manner of Turner and Weiss (1963) are the same as those obtained from the new definition. Thus, the new definition subsumes rather than replaces the old one. Such folds are special in the sense that they possess strong hinges but no weak ones. Another special feature of such folds is that the hinge lines are parallel with principal curvature trajectories. In the general case, fold hinges are not themselves curvature trajectories (Koenderink, 1990, p. 292). However, any line of symmetry in the surface is both a principal curvature trajectory and a fold hinge line (Porteous, 1994, p. 162).

5. Methods of constructing hinge lines

Current interest within a number of non-geological disciplines in lines of extremal curvature on surfaces has resulted in a large number of algorithms of potential application for the computation of the hinge lines on folded surfaces, e.g. Yoshizawa et al. (2008) and papers cited therein. Discussion of different approaches is beyond the present remit. Instead, we outline two alternative strategies to demonstrate the feasibility of this type of analysis.

For both of the methods described, input data are required that describe the location of points on the surface to be analyzed. The data format corresponds to x, y, z coordinates of these points, where z is the height (z) of the surface at a given different points x, y across a map. Data of this kind can be obtained from seismic mapping software, from high precision GPS surveying of exposed folded surfaces (e.g. Xu et al., 2000; Pearce et al., 2006) or from 3D laser scanning of outcrops or hand specimens (Buckley et al., 2008). These data are used to compute principal curvatures at points across the surface. Different algorithms have been proposed for the calculation of principal curvatures for geological surfaces (Lisle and Robinson, 1995; Samson and Mallett, 1997; Ozkaya, 2002; Bergbauer et al., 2003; Bergbauer and Pollard, 2003; Pearce et al., 2006). Although some software for seismic interpretation calculates some curvature properties these rarely include principal curvatures.

Our estimated principal curvatures were calculated from data points arranged in a square-grid using a FORTRAN 95 program based on the procedures laid out by Bergbauer and Pollard (2003). The program estimates principal curvatures at grid nodes, but does not have the capability for drawing the principal curvature trajectories across the surface. It is anticipated that the latter capability (e.g. Kalogerakis et al., 2009) will eventually simplify the drawing of fold hinge lines on geological surfaces. The two methods described below were chosen because they require only gridded curvature values.

5.1. Automatic method

This strategy is based on the algorithm of Belyaev and Anoshkina (2005). This approach seeks to identify hinge points by comparing the principal curvature value of each data grid node, P, with the principal curvatures at its eight neighbours (Fig. 5). The aim is to consider whether the absolute magnitude of the principal curvature at P is extremal, i.e. whether the magnitude at P exceeds the magnitudes at P_A and P_B, where P_A and P_B are located on the perimeter of the box defined by the eight neighbours and lie on the line P_A–P–P_B which is parallel to the corresponding principal curvature direction at P. Since P_A and P_B are not grid points, their curvature values have to be estimated by bilinear interpolation (Belyaev and Anoshkina, 2005). The point P is considered to be a hinge point if the curvature at P exceeds that at P_A and at P_B (Fig. 5).

Since computing hinge lines involves estimation of high-order surface derivatives, these surface features are very sensitive to noise. This method in its basic form is adversely affected by noise. The result is an abundance of hinge points distributed diffusely rather than along curvilinear hinge lines. Several authors have suggested procedures that may help alleviate this problem of noisy data, e.g. Stewart and Wynn (2000), Bergbauer et al. (2003), Belyaev and Anoshkina (2005), Kim and Kim (2005) and Yoshizawa et al. (2008).

5.2. Semi-automatic method

This strategy is an intuitive procedure that requires manual input during the selection of hinge points and the drawing of hinge lines. As with the previous method, the data required for defining the folded surface consist of the coordinates (x, y, z) of points arranged on a regular grid (Fig. 6). At each grid node the principal curvatures, k_1 and k_2, as well as their directions, are computed (e.g. Bergbauer and Pollard, 2003). The two principal curvatures are then ordered in terms of their absolute magnitudes, $|k|_{max}$ and $|k|_{min}$, and their variation across the surface displayed by means of curvature isoline maps. The corresponding principal curvature directions are used to construct the direction field maps.

Strong hinge lines are constructed by overlaying the curvature isoline map of $|k|_{max}$ and the direction field map of $|k|_{max}$ (Fig. 5b). Hinge points are located by visually tracking along the curvature trajectories, i.e. traversing whilst following the direction field, until a local maximum of $|k|_{max}$ is found (point H in Fig. 6b). These hinge points, when joined together, form the hinge line (Fig. 6c). Weak hinge lines are found in the same manner but by using the isoline and direction field maps of $|k|_{min}$.

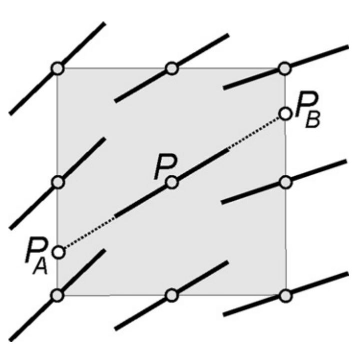

Fig. 5. Principle of an automatic method of detecting hinge points from an array of grid points with calculated principal curvatures, k. P is a hinge point with k at P exceeds k at points P_A and P_B, where the line P_A–P–P_B is parallel to the principal curvature direction at P, and where the principal curvatures at P_A and P_B are estimated by linear interpolation between neighbouring points.

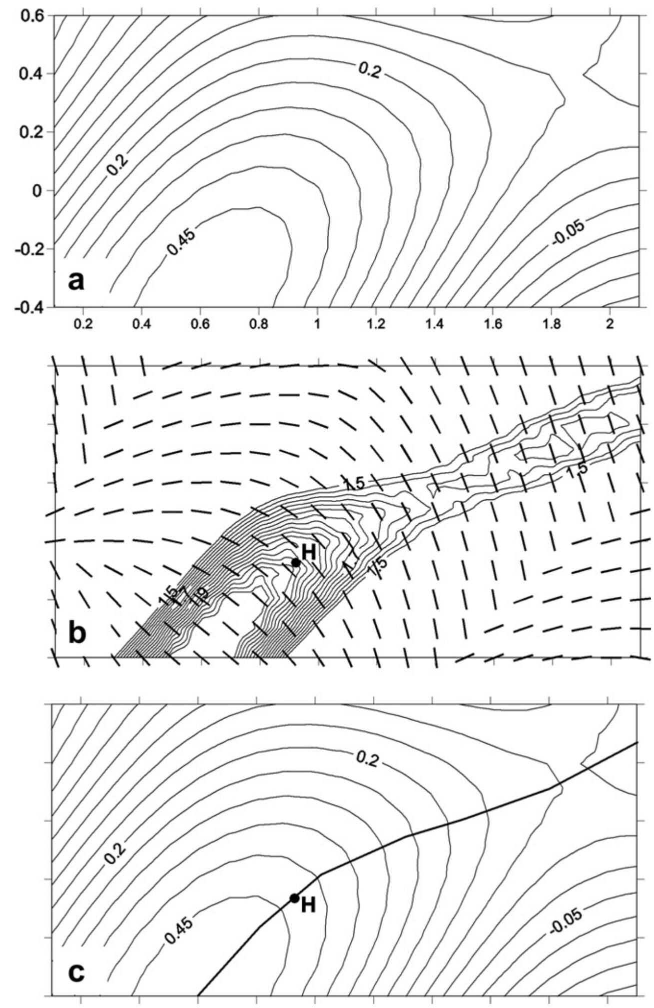

Fig. 6. Method for drawing hinge lines. (a) A simulated folded surface represented by structure contours. (b) Principal curvature with maximum absolute value; directions shown as dashes, absolute magnitude shown as contours; H is a hinge point. (c) Structure contours and constructed hinge line (strong hinge).

The semi-automatic method is applied to an example of a seismically mapped folded horizon of Kimmeridgian age, off-shore northern Spain (confidential commercial data). Fig. 7 is a map showing structure contours, antiformal and synformal folds bounded by the contour of zero mean curvature (Lisle and Toimil, 2007) and strong hinge lines. This map illustrates the fact that hinge lines are not reliably discernable simply from inspection of the curvature of the structure contours. For example, points of maximum curvature of the structure contours do not generally lie on hinge lines. This is because the form of structure contours is governed only by curvature of the surface in a horizontal plane, whereas the hinge line is related to principal curvatures. Secondly, this example reminds us that the hinge lines do not necessarily coincide with the crestal portions of folds where the fold reaches maximum elevation. We also note that hinge lines are not continuous linear features, but are themselves curved and can loose their identity and disappear.

In the southern part of the map, the hinge lines define an orthogonal pattern indicating two perpendicular fold directions and a dome-and-basin structure. In general the antiformal areas, areas where the mean curvature is positive, tend to be blob-like in map view. The hinge lines with the synformal regions often wrap around the antiformal areas, suggesting a rim-syncline geometry around diapiric antiforms.

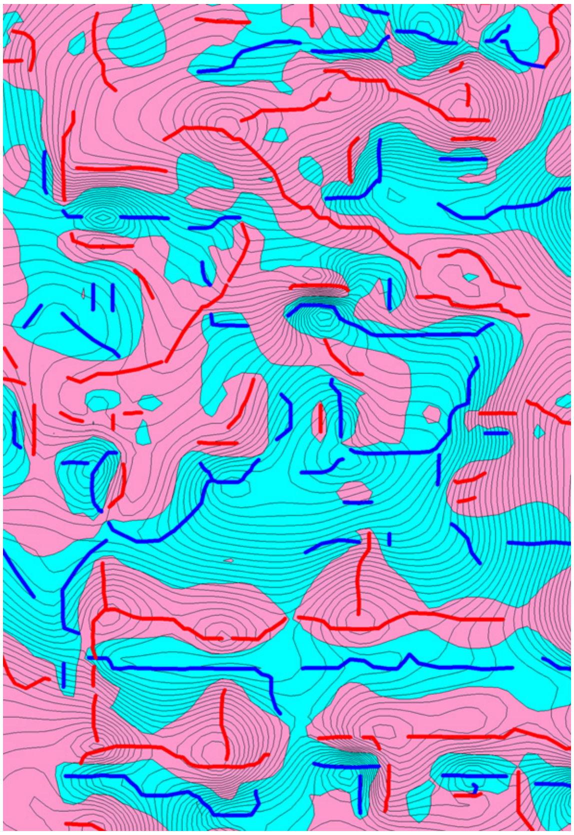

Fig. 7. Hinge line map, seismically mapped horizon, Kimmeridgian, Cantabrian-Basque Basin, N. Spain. Structure contours, thin lines; strong hinge lines, thick lines. Antiforms (red) and synforms (blue) are bounded by the zero mean curvature contour (Lisle and Toimil, 2007). Width of map 25 km.

6. Conclusions

Given the existence of more 3D data sets from structural surfaces and the computational advances for describing and representing those surfaces, the need for efficient accurate techniques for geometrical analysis of the acquired data increases.

An established technique for structural analysis is the construction of hinge lines to portray the major undulations of the folded surfaces. However, existing definitions of the hinge line are inappropriate for use with folds of non-cylindrical shape. We therefore propose a revised definition with general application, based on the concept of ridge lines used in the field of differential geometry. Further work is required to develop efficient strategies for the automatic identification of hinge lines on geological surfaces.

Hinge lines cannot be mapped by inspection of the structure contour pattern, making computation necessary. For example, points of maximum curvature of structure contours do not generally lie on a hinge line. Furthermore, closed loops of the contours indicating a local maximum or minimum altitude of the surface do not necessary contain part of the hinge line.

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